The Decomposition of a Tensor in a Sum of Rank-(R_1, R_2, R_3) Terms

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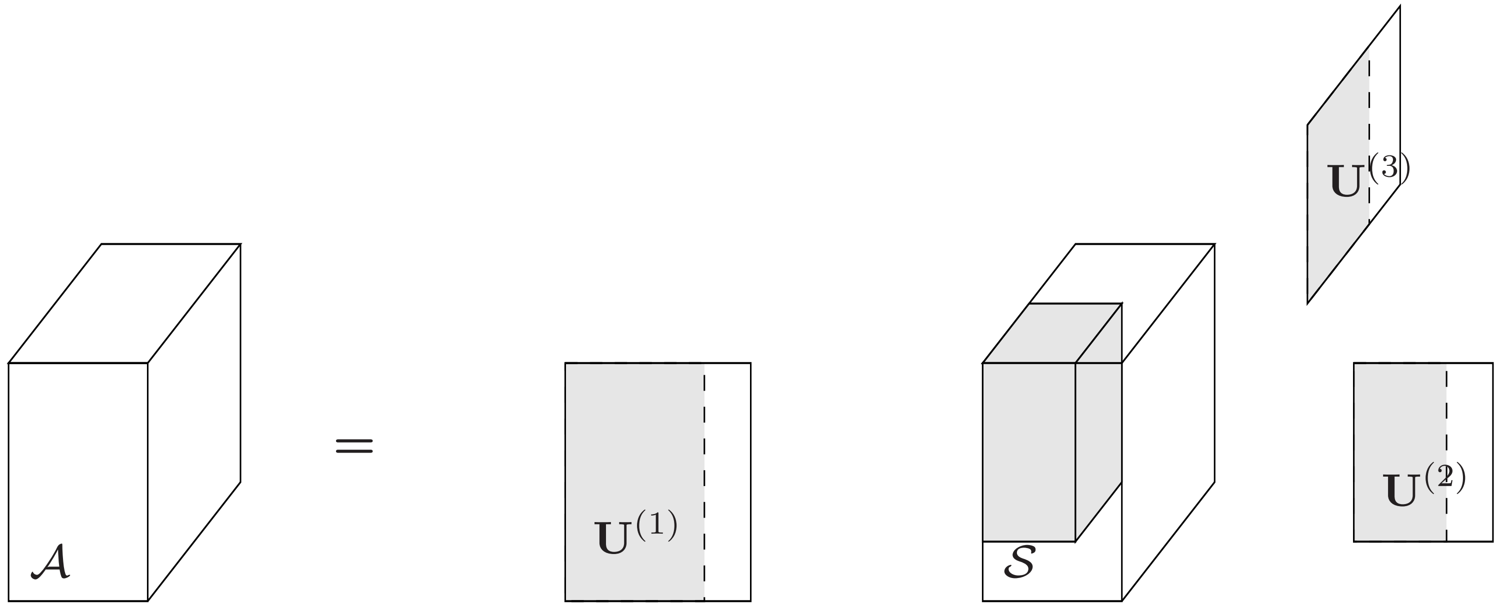
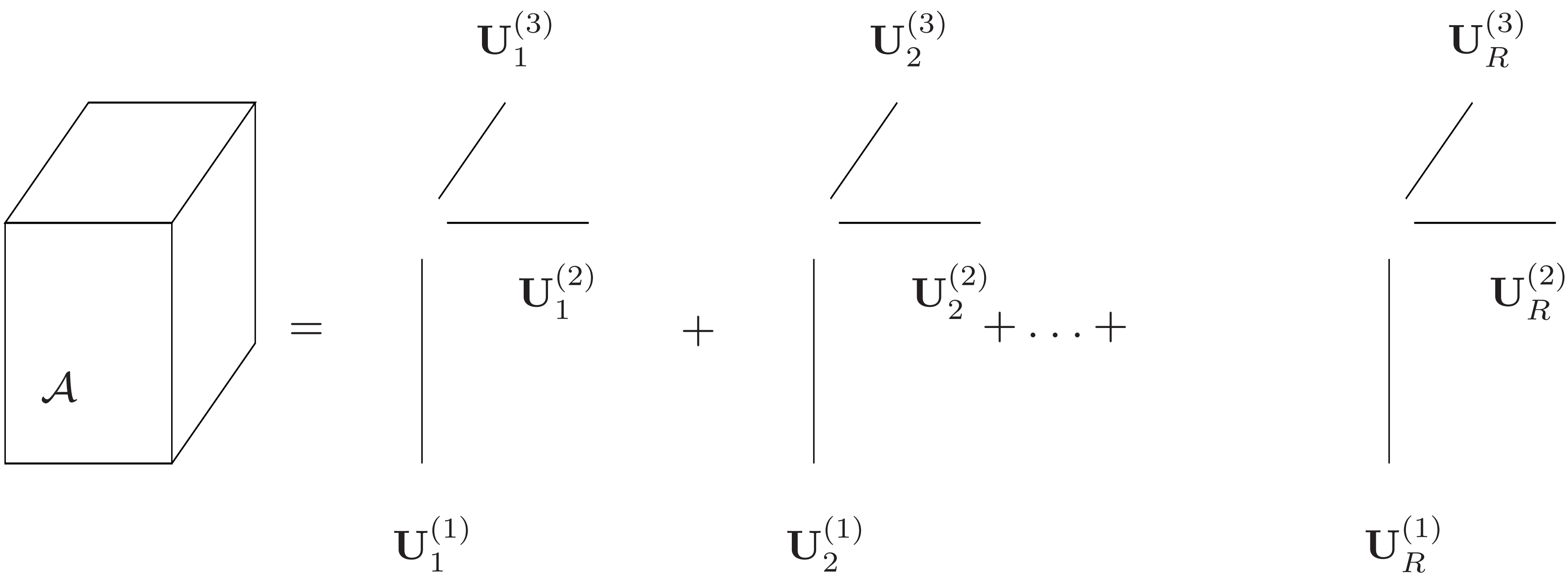
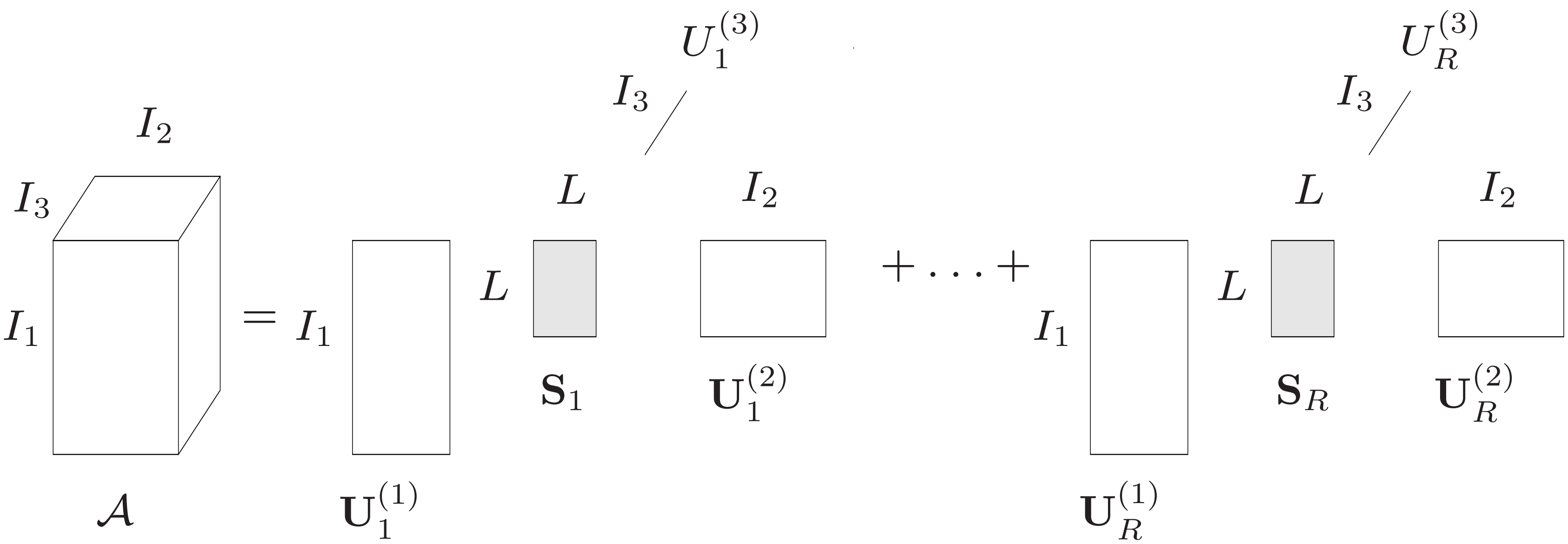
Tucker's decomposition and best rank-(R_1, R_2, R_3) approximation



PARAFAC


$\mathcal{A}$
$=$
$\mathbf{U}_1^{(3)}$
$\mathbf{U}_1^{(2)}$
$\mathbf{U}_1^{(1)}$
$+$
$\mathbf{U}_2^{(3)}$
$\mathbf{U}_2^{(2)}$
$\mathbf{U}_2^{(1)}$
$+ \ldots +$
$\mathbf{U}_R^{(3)}$
$\mathbf{U}_R^{(2)}$
$\mathbf{U}_R^{(1)}$

Decomposition in rank-$(L, L, 1)$ terms

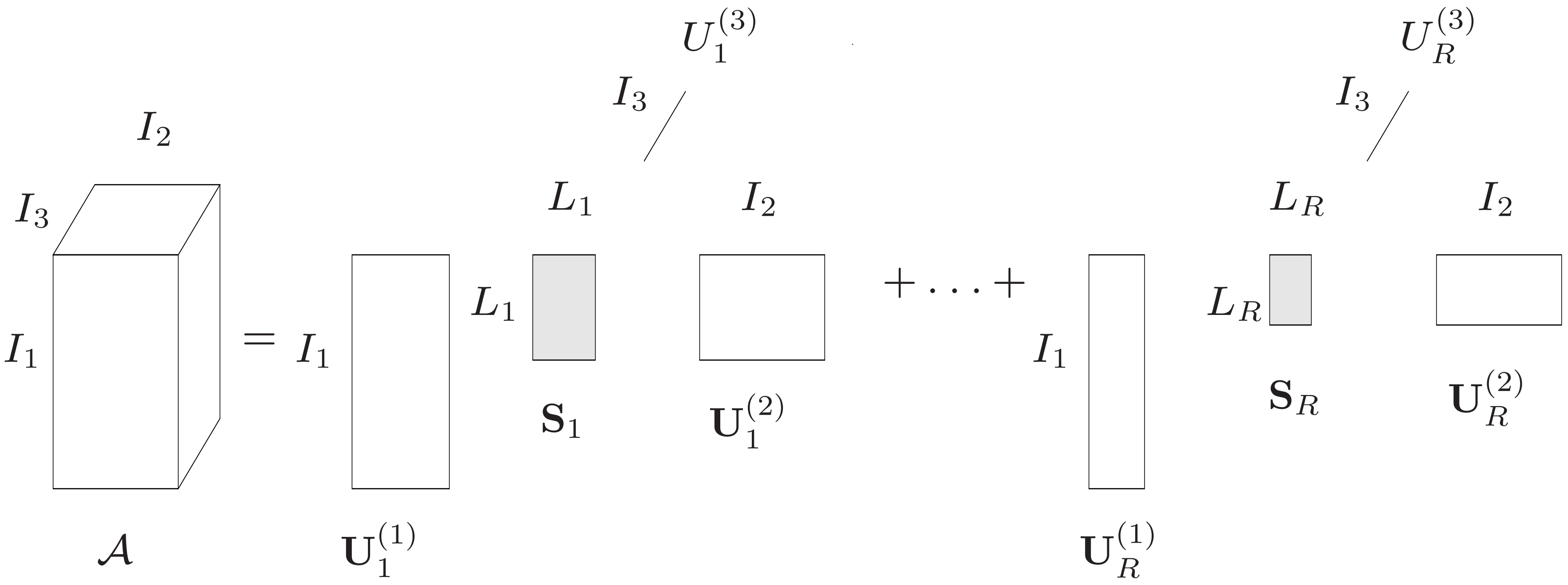


Uniqueness

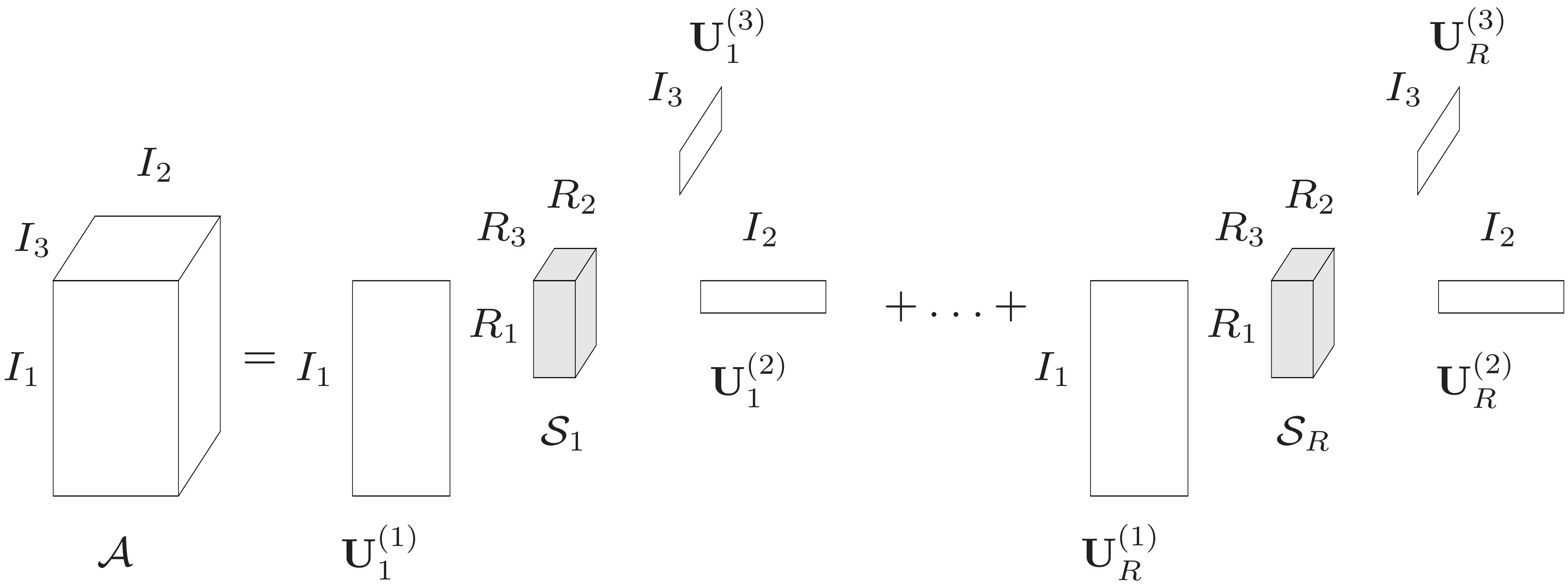
$$\min\left(\left\lfloor \frac{I_1}{L} \right\rfloor, R\right) + \min\left(\left\lfloor \frac{I_2}{L} \right\rfloor, R\right) + \min(I_3, R) \geqslant 2R + 2$$

cf. $\min(I_1, R) + \min(I_2, R) + \min(I_3, R) \geqslant 2R + 2$ (PARAFAC)

Decomposition in rank-$(L_r, L_r, 1)$ terms



Decomposition in rank-(R_1, R_2, R_3) terms



A decomposition structure

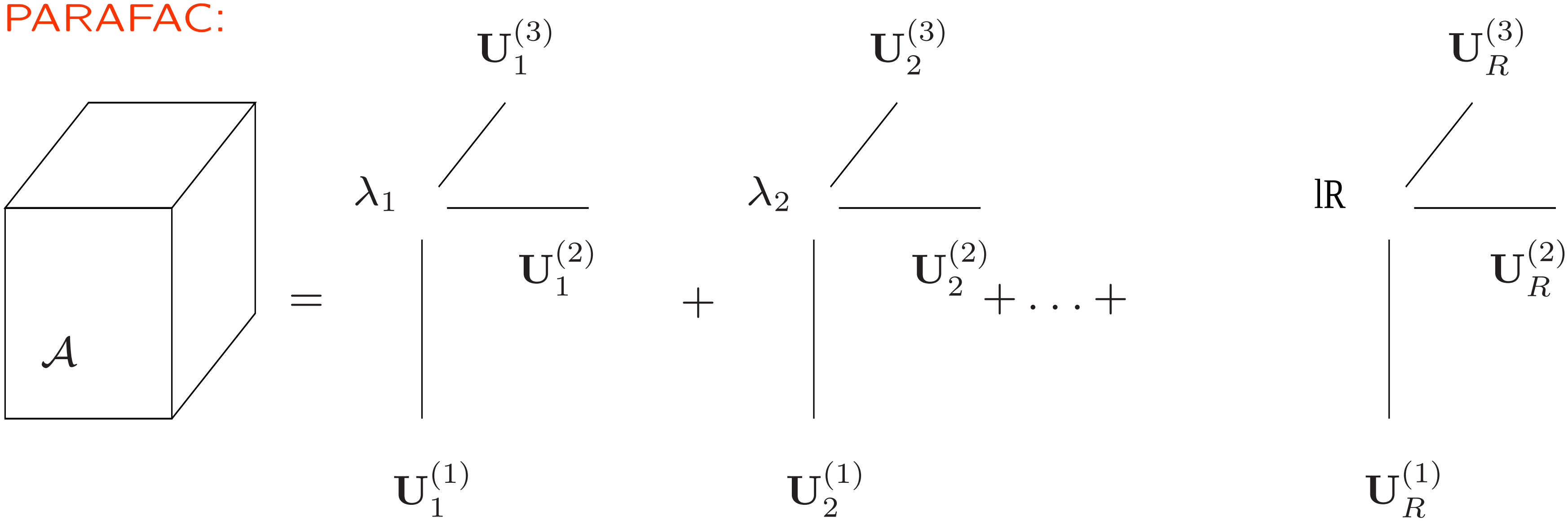
$(I_1 \times I_2 \times I_3)$ tensor $\mathcal{A}$:

$$
\left|
\begin{array}{c|c|c|c}
(I_1, I_2, I_3) & 2(I_1, I_2, \lceil I_3/2 \rceil) & & R(1) \\
(I_1, I_2, I_3 - 1) & 2(I_1, I_2, \lfloor I_3/2 \rfloor) & & \\
(I_1 - 1, I_2 - 2, I_3 - 3) & (I_1, I_2, \lfloor I_3/2 \rfloor) + (I_1, \lfloor I_2/2 \rfloor, I_3) & \cdots & \\
\vdots & \vdots & & \\
1 & 2(1) & &
\end{array}
\right|
$$

- rank
- generalization SVD
- typical rank
- degeneracy
- complex factors cf. [*Kaporin, '05*]

ALS algorithm

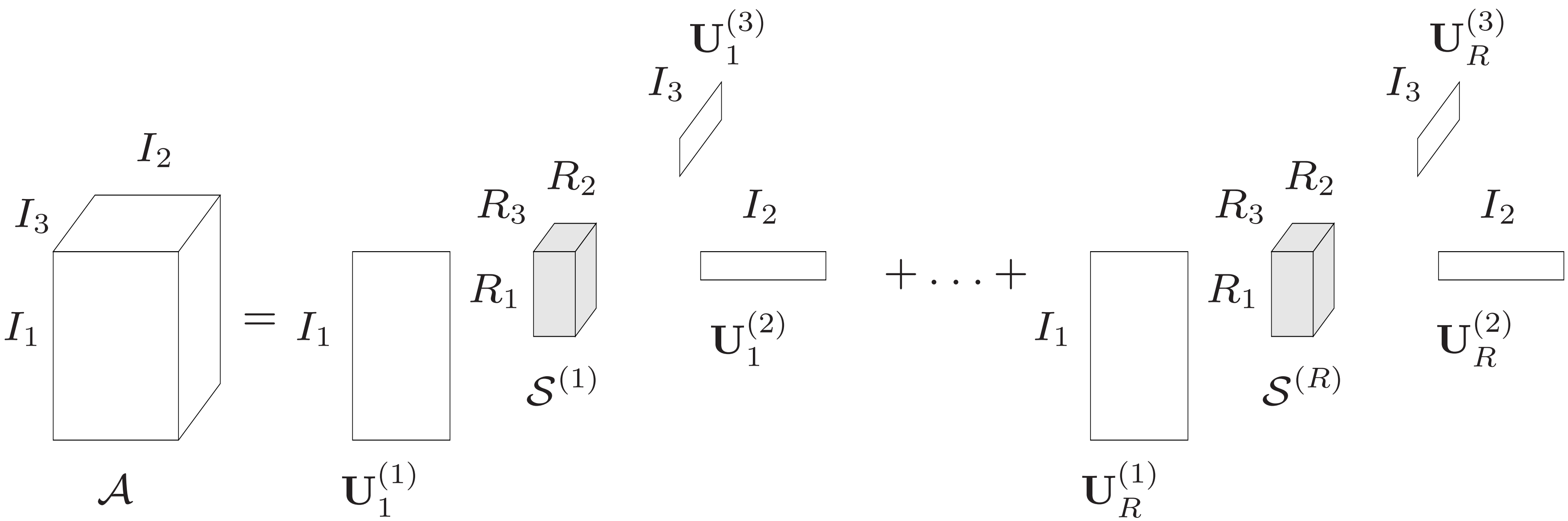
PARAFAC:



$$\mathbf{M}(\{U_r^{(2)}\}, \{U_r^{(3)}\})X = A$$

$$x(1 : I_1) \sim \lambda_1 U_1^{(1)}$$

Decomposition in rank-(R_1, R_2, R_3) terms:



$$\mathbf{M}(\{\mathbf{U}_r^{(2)}\}, \{\mathbf{U}_r^{(3)}\})X = A$$

$$x(1 : I_1 R_2 R_3) \quad \sim \quad \mathbf{U}_1^{(1)} \cdot \mathbf{S}^{(1)}$$

$$(I_1 \times R_1)(R_1 \times R_2 R_3)$$

Perspectives

- Simultaneous matrix decompositions
- Enhanced line search
- Levenberg-Marquardt
- Model order and model structure selection